The Logistic Equation

The weather is chaotic. This means that unless we know perfectly what the weather is doing at a given time (down to the scale of mm), we can not make a perfect weather forecast. Small differences in the current conditions can lead to big differences in how the weather develops. There are other, simpler situations where a small change in the starting conditions can lead to a big change in the result.

Where are most weather observations made? Where is the weather observed infrequently?

Consider a quantity, $x$, whose current value is known. If we also know how it evolves in time, we can say what the value of $x$ will be a short time $\Delta t$ later. If we call the current time $t_1$, then we could call $t+\Delta t=t_2$, $t_2+\Delta t=t_3$ etc. In other words, we can use the value of $x$ at any time $t$, $x_t$, to calculate its value $\Delta t$ later, $x_{t+1}$.

The Logistic equation is one particular way $x$ could evolve.

It is written

$$x_{t+1} = k x_t (1-x_t)$$

This means each subsequent value of $x$ depends on the previous value.

Open the Excel file ‘Logistic_equation.xls’.

In the spreadsheet, the values of $k$ and $x_{t=0}$ are set at the top.

The table allows you to see the value of $x$ over 60 iterations

**How does $x$ change with time?**

Try values of $k$ between 2.4 and 3.5.

**What do you notice happening?**
At what value of \( k \) does the behaviour of the system go from being 'normal' to being 'chaotic'? 

Choose a value of \( k \) in the normal region, and vary \( x_{t=0} \) slightly (e.g. from 0.2 to 0.21) 

How does \( x \) change with time? 

Choose a value of \( k \) in the chaotic region, and vary \( x_{t=0} \) slightly (e.g. from 0.2 to 0.21) 

How does \( x \) change with time? 

Chaos is when a small change in initial conditions can lead to big differences in the behaviour of the system. 

How does our imperfect knowledge of the current weather lead to poor weather forecasts?