A very simple energy-balance climate model

The temperature of the Earth depends on the difference between the amount of energy the Earth receives from the Sun, and the amount of energy the Earth loses to space. For the Earth’s average temperature to be steady, energy must leave at the same rate as it arrives. If energy arrives faster than it leaves, the Earth’s temperature will rise.

Question 1
- Is the Earth’s temperature constant?
- What might cause the Earth’s temperature to change? Suggest as many factors as possible which might cause the Earth’s temperature to increase or decrease.

Gaining and losing energy
On average, the rate at which radiation reaches the Earth’s surface is 364 $\text{Js}^{-1}\text{m}^{-2}$ (that is, 364 J of energy arrives on every square metre of the Earth’s surface every second).

The rate at which the Earth radiates energy back into space depends only on the temperature of the Earth, and is given by the Stefan-Boltzmann Law:

\[
\text{rate of outgoing radiation per square metre} = \sigma T^4
\]

where $\sigma = 5.67 \times 10^{-8} \text{Js}^{-1}\text{m}^{-2}\text{K}^{-4}$ (the Stefan-Boltzmann constant)

Calculation 1
Let’s start by assuming that the average temperature of the Earth’s surface is 283 K.
- Calculate the rate at which energy leaves the Earth.
- What do you notice about this value?
- Will the Earth’s temperature increase, decrease, or stay the same?
**Warming up**

The change in temperature of the Earth depends on the difference between the rate at which radiation arrives and the rate at which it leaves. It also depends on the heat capacity of the Earth (how easily the Earth heats up).

The heat capacity of the Earth = $4 \times 10^8 \text{ JK}^{-1}\text{m}^{-2}$; i.e. it takes $4 \times 10^8 \text{ J}$ to raise the temperature of $1 \text{ m}^2$ by $1 \text{ K}$.

The change in temperature of the Earth over one year is given by:

$$\text{temperature change} = (\text{incoming radiation} – \text{outgoing radiation}) \times \frac{\text{time}}{\text{heat capacity}}$$

and the temperature of the Earth after one year will be given by:

$$\text{new temperature} = \text{old temperature} + \text{temperature change}$$

**Calculation 2**

Consider the case where the incoming energy increases by 5%.

- Show that the rate of incoming radiation is now $382.2 \text{ Js}^{-1}\text{m}^{-2}$.
- Calculate the net inflow of radiation, (incoming radiation – outgoing radiation).
- Calculate the increase in the Earth’s temperature after 1 year.

**Approaching equilibrium**

After one year, the Earth will be warmer, so it will radiate faster. In Year 2, its temperature will rise again, but less than during Year 1. In Year 3, its temperature will rise again, but by a smaller amount. Eventually, it will reach a steady state.

**Question 2**

- Explain why the temperature rise in Year 2 is smaller than that in Year 1.
- What can you say about the rates of radiation when the Earth’s temperature stabilises?
- Sketch a graph to show how you think the Earth’s temperature will change as it approaches the steady state.

**Calculation 3**

It is easier to perform an iterative calculation like this using a table.

- Complete the table (next page).
- Plot a graph of temperature against time. Is a new steady state reached, or is the Earth’s temperature unstable, rapidly increasing or decreasing?
<table>
<thead>
<tr>
<th>Time / years</th>
<th>Incoming radiation / J s⁻¹ m⁻²</th>
<th>Outgoing radiation / J s⁻¹ m⁻²</th>
<th>Change in temperature / K</th>
<th>Temperature of Earth / K</th>
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</table>

**Thinking about the model**

We use models in science because they can help us to understand real situations and systems, and because they can help us to make predictions. However, all models have their limitations.

The Sun, Earth and atmosphere are a very complex system; our model is very simple. Our model assumes that we can represent the Earth by an average square metre of its surface. It also assumes that radiation arrives from the Sun at a steady rate, which suddenly changes.

**Question 3**

- In what ways is our model unrealistic?
- How might it be improved?

**Question 4**

Your calculations determine the Earth’s new temperature at the end of each year. They assume that the Earth’s temperature is constant during the year, when it is actually rising.

- How could you improve the method of calculation?
- How could you automate this model using a computer spreadsheet?

[climateprediction.net](http://climateprediction.net)